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Cadeira: CONTINUUM MECHANICS **Época: Normal**

School year: 2018/2019 (1st Semestre) **TEST 1** (2018/11/03) **Duration:** 1,5 hours

Name: _____Number:_____ Course: **EC**

Questions 1 to 6 should be answered in this sheet. Please do not write derivations or calculations in these questions; they will not be rated anyway. It is not worth presenting several variants of the answer (if they are contradictory, the answer will be considered incorrect). The remaining questions should be answered on the examination sheets with appropriate derivations and calculations.

1. [2]

   (a) Evaluate the mean free path of air molecules under normal conditions. Reference information: the number density of air at 20 °C is 2.46×10^{19} moléculas/ cm^3, the collision cross section of molecules of air is approximately $10^{-15}\,\mathrm{cm}^2$.

   Answer ___

   (b) The continuity hypothesis can be used to study the motion of air around a microstructure with the characteristic dimension of 100 nm under normal conditions? Note: the answer should be based on the result obtained in the previous paragraph.

2. [2] Rewrite the following formula in a complete form (i.e., without using abbreviations):

$$a_i = U_{im} V_{mk} c_k$$

   Answer:

3. [2] Consider five tensors $\widehat{Q_1}$, $\widehat{Q_2}$, ..., $\widehat{Q_5}$. It is known that

$$\begin{aligned}
\widehat{Q_1}\mathbf{e}_1 &= \mathbf{e}_2, \quad \widehat{Q_1}\mathbf{e}_2 = (\mathbf{e}_1 - \mathbf{e}_3)/2 \\
\widehat{Q_2}\mathbf{e}_1 &= (\mathbf{e}_1 + \mathbf{e}_2)/\sqrt{2}, \quad \widehat{Q_2}\mathbf{e}_2 = (\mathbf{e}_2 - \mathbf{e}_3)/\sqrt{3} \\
\widehat{Q_3}\mathbf{e}_1 &= (\mathbf{e}_1 + \mathbf{e}_2)/\sqrt{2}, \quad \widehat{Q_3}\mathbf{e}_2 = (\mathbf{e}_2 - \mathbf{e}_1)/\sqrt{2} \\
\widehat{Q_4}\mathbf{e}_1 &= (3\mathbf{e}_1 + 4\mathbf{e}_2)/5, \quad \widehat{Q_4}\mathbf{e}_2 = (\mathbf{e}_2 - 3\mathbf{e}_3)/\sqrt{10} \\
\widehat{Q_5}\mathbf{e}_1 &= (\mathbf{e}_1 + 2\mathbf{e}_2)/\sqrt{5}, \quad \widehat{Q_5}\mathbf{e}_2 = \mathbf{e}_1
\end{aligned}$$

   Which ones of these tensors can be orthogonal?

   Answer ___

4. [2] Consider a tensor $\widehat{T}$ associated with the matrix $\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and vectorial function $\mathbf{a}(\mathbf{r}) = (3x_1, 6, 9)$. Find:

   (a) Gradient of the function $\mathbf{a}(\mathbf{r})$.
   Answer:

   (b) $\text{tr}\left[\widetilde{\widehat{T}}(\nabla \mathbf{a})\right]$.
   Answer:

5. [2] The density and velocity fields in a continuum in dimensionless variables are

$$\rho = \rho(x_1, x_2, x_3, t) = a\exp(-2x_1x_2) + bx_3 + ct, \qquad \mathbf{v} = \mathbf{v}(x_1, x_2, x_3, t) = (x_2, \cos x_2,\ fx_1),$$

   where a, b, c e f are given constants. What is the material derivative of density?
   Answer

6. [2]

   (a) Write the compatibility condition for components of the infinitesimal strain tensor that involve components E_{22}, E_{33} and E_{23}.
   Answer:

   (b) Consider the infinitesimal strain tensor with the components $E_{11} = E_{12} = E_{13} = 0$, $E_{22} = 2X_3^2$, $E_{33} = 6X_2X_3^2$, $E_{23} = kX_2X_3 + 3X_3^2 + X_3^3 + 2$, where k is a constant. It is known that this tensor is compatible in a simply connected region. Determine the constant k.
   Answer:

7. [3] Find the eigenvalues and eigenvectors of the tensor of right hand rotation of 45° about $\mathbf{e}_3$.

8. [5] Given the displacement field

$$u_1 = k\left(X_2^2 - X_1^2\right); \quad u_2 = k\left(2X_2 + X_1^2\right); \quad u_3 = 0; \quad k = 10^{-4}$$

   (a) Find the unit elongation of and the change of angle between the material elements $d\mathbf{X}_1 = dX_1\mathbf{e}_1$ and $d\mathbf{X}_2 = dX_2\mathbf{e}_2$, which emanate from a particle at the postion $\mathbf{X} = -\mathbf{e}_1 + \mathbf{e}_2$.

   (b) Find these material elements after the deformation.